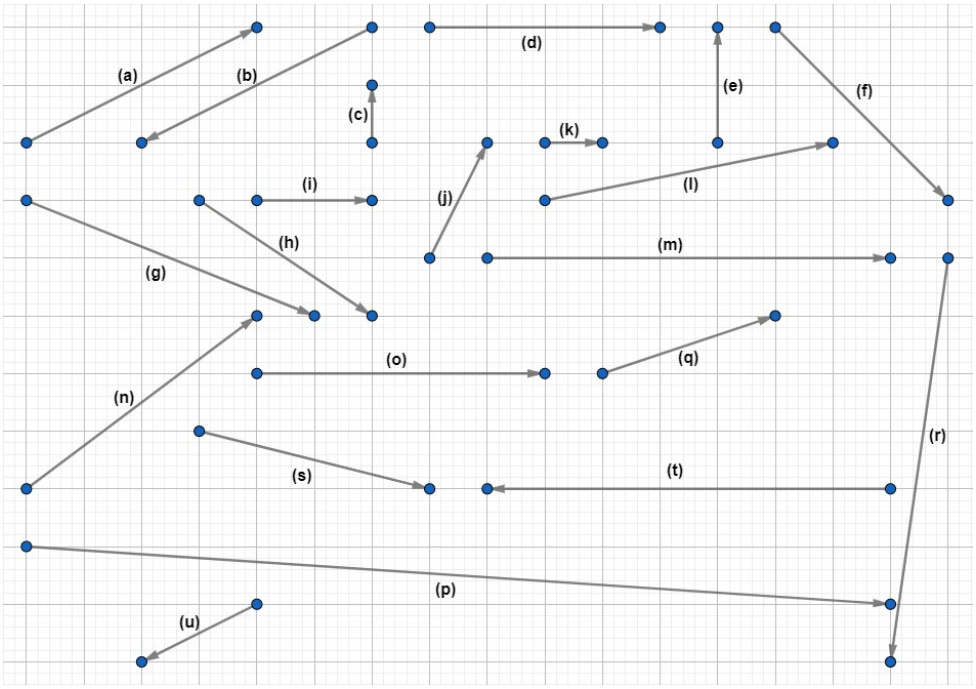
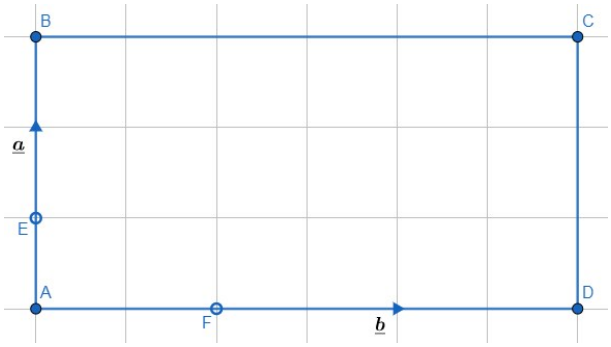
Revision Exercise (Vectors)

1.



2.

(i)



(ii) $\overrightarrow{BD} = \underline{b} - \underline{a}$ and $\overrightarrow{EF} = \frac{1}{3}(\underline{b} - \underline{a}) = \frac{1}{3}\overrightarrow{BD}$, so EF is parallel to BD.

(iii) a) $\overrightarrow{AC} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

(iii) b) $|\overrightarrow{AC}| = \sqrt{26}$ units

(iii) c) $\widehat{\overrightarrow{AC}} = \begin{pmatrix} \frac{5}{\sqrt{26}} \\ \frac{1}{\sqrt{26}} \end{pmatrix}$

3.

(i) $\overrightarrow{PQ} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ $\overrightarrow{SR} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ $\overrightarrow{SP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\overrightarrow{RQ} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(ii) PQRS is a parallelogram.

(iii) $\overrightarrow{PS} \cdot \overrightarrow{PQ} = -17$ $\left|\overrightarrow{PS}\right| = \sqrt{13}$ $\left|\overrightarrow{PQ}\right| = \sqrt{50}$

(iv) $\overrightarrow{QPS} = 131.82°$

4.

a) (i) -68	(ii) $\lvert\underline{a}\rvert = 2\sqrt{17}$	$\lvert\underline{b}\rvert = 2\sqrt{34}$	(iii) 135°	(iv) Obtuse
b) (i) 34	(ii) $\lvert\underline{a}\rvert = \sqrt{17}$	$\lvert\underline{b}\rvert = 2\sqrt{17}$	(iii) 0°	(iv) Straight
c) (i) 4	(ii) $\lvert\underline{a}\rvert = \sqrt{5}$	$\lvert\underline{b}\rvert = \sqrt{5}$	(iii) 36.9°	(iv) Acute
d) (i) -1	(ii) $\lvert\underline{a}\rvert = \sqrt{10}$	$\lvert\underline{b}\rvert = \sqrt{17}$	(iii) 94.4°	(iv) Obtuse
e) (i) 11	(ii) $\lvert\underline{a}\rvert = \sqrt{5}$	$\lvert\underline{b}\rvert = 5$	(iii) 10.3°	(iv) Acute
f) (i) 10	(ii) $\lvert\underline{a}\rvert = \sqrt{5}$	$\lvert\underline{b}\rvert = 2\sqrt{5}$	(iii) 0°	(iv) Straight
g) (i) -7	(ii) $\lvert\underline{a}\rvert = \sqrt{34}$	$\lvert\underline{b}\rvert = \sqrt{5}$	(iii) 122.5°	(iv) Obtuse
h) (i) 12	(ii) $\lvert\underline{a}\rvert = 4$	$\lvert\underline{b}\rvert = \sqrt{10}$	(iii) 18.4°	(iv) Acute
i) (i) 0	(ii) $\lvert\underline{a}\rvert = 3$	$\lvert\underline{b}\rvert = 5$	(iii) 90°	(iv) Right
j) (i) 218	(ii) $\lvert\underline{a}\rvert = \sqrt{109}$	$\lvert\underline{b}\rvert = 2\sqrt{109}$	(iii) 0°	(iv) Straight
k) (i) 0	(ii) $\lvert\underline{a}\rvert = \sqrt{29}$	$\lvert\underline{b}\rvert = \sqrt{29}$	(iii) 90°	(iv) Right

l) (i) -46 (ii) $|\underline{a}| = \sqrt{58}$ $|\underline{b}| = 5\sqrt{2}$ (iii) 148.7° (iv) Obtuse

m) (i) 28 (ii) $|\underline{a}| = 2\sqrt{10}$ $|\underline{b}| = \sqrt{34}$ (iii) 40.6° (iv) Acute

n) (i) 14 (ii) $|\underline{a}| = 5$ $|\underline{b}| = 2\sqrt{29}$ (iii) 74.9° (iv) Acute

o) (i) 4 (ii) $|\underline{a}| = \frac{1}{2}\sqrt{2}$ $|\underline{b}| = 4\sqrt{2}$ (iii) 0° (iv) Straight

p) (i) $-\frac{3}{20}$ (ii) $|\underline{a}| = \frac{1}{10}\sqrt{26}$ $|\underline{b}| = \frac{3}{10}\sqrt{109}$ (iii) 95.4° (iv) Obtuse

5.

a)

(i) $|\underline{u}| = 5$
(ii) $|\underline{v}| = \sqrt{13}$
(iii) $\underline{\widehat{u}} = \frac{4}{5}\widehat{i} + \frac{3}{5}\widehat{j}$
(iv) $\underline{\widehat{v}} = \frac{3}{\sqrt{13}}\widehat{i} + \frac{2}{\sqrt{13}}\widehat{j}$
(v) 18
(vi) 0.055 rad

b)

(i) $|\underline{u}| = 2\sqrt{26}$
(ii) $|\underline{v}| = \sqrt{13}$
(iii) $\underline{\widehat{u}} = -\frac{1}{\sqrt{26}}\widehat{i} - \frac{5}{\sqrt{26}}\widehat{j}$
(iv) $\underline{\widehat{v}} = \frac{2}{\sqrt{13}}\widehat{i} + \frac{3}{\sqrt{13}}\widehat{j}$
(v) -34
(vi) 2.75 rad

c)

(i) $|\underline{u}| = 5$
(ii) $|\underline{v}| = 12$
(iii) $\underline{\widehat{u}} = \widehat{i}$
(iv) $\underline{\widehat{v}} = \widehat{i}$
(v) 60
(vi) 0 rad

d)

(i) $|\underline{u}| = \sqrt{13}$
(ii) $|\underline{v}| = \sqrt{13}$
(iii) $\underline{\widehat{u}} = \frac{2}{\sqrt{13}}\widehat{i} + \frac{3}{\sqrt{13}}\widehat{j}$
(iv) $\underline{\widehat{v}} = -\frac{2}{\sqrt{13}}\widehat{i} - \frac{3}{\sqrt{13}}\widehat{j}$
(v) -13
(vi) π rad

e)

(i) $|\underline{u}| = 4$
(ii) $|\underline{v}| = 5$
(iii) $\underline{\widehat{u}} = \widehat{j}$
(iv) $\underline{\widehat{v}} = \widehat{i}$
(v) 0
(vi) $\frac{\pi}{2}$ rad

f)

(i) $|\underline{u}| = 2$
(ii) $|\underline{v}| = \sqrt{2}$
(iii) $\widehat{\underline{u}} = -\widehat{j}$
(iv) $\widehat{\underline{v}} = \frac{1}{\sqrt{2}}\widehat{i} + \frac{1}{\sqrt{2}}\widehat{j}$
(v) -2
(vi) 2.36 rad

g)

(i) $|\underline{u}| = 3$
(ii) $|\underline{v}| = \sqrt{2}$
(iii) $\widehat{\underline{u}} = -\widehat{i}$
(iv) $\widehat{\underline{v}} = \frac{1}{\sqrt{2}}\widehat{i} - \frac{1}{\sqrt{2}}\widehat{j}$
(v) -3
(vi) 2.36 rad

h)

(i) $|\underline{u}| = 2\sqrt{5}$
(ii) $|\underline{v}| = \sqrt{10}$
(iii) $\widehat{\underline{u}} = \frac{2}{\sqrt{5}}\widehat{i} - \frac{1}{\sqrt{5}}\widehat{j}$
(iv) $\widehat{\underline{v}} = \frac{3}{\sqrt{10}}\widehat{i} + \frac{1}{\sqrt{10}}\widehat{j}$
(v) 10
(vi) 0.79 rad

i)

(i) $|\underline{u}| = 2\sqrt{5}$
(ii) $|\underline{v}| = \sqrt{29}$
(iii) $\widehat{\underline{u}} = -\frac{1}{\sqrt{5}}\widehat{i} + \frac{2}{\sqrt{5}}\widehat{j}$
(iv) $\widehat{\underline{v}} = \frac{5}{\sqrt{29}}\widehat{i} + \frac{2}{\sqrt{29}}\widehat{j}$
(v) -2
(vi) 1.65 rad

j)

(i) $|\underline{u}| = 3\sqrt{2}$
(ii) $|\underline{v}| = 4\sqrt{2}$
(iii) $\widehat{\underline{u}} = -\frac{1}{\sqrt{2}}\widehat{i} + \frac{1}{\sqrt{2}}\widehat{j}$
(iv) $\widehat{\underline{v}} = \frac{1}{\sqrt{2}}\widehat{i} + \frac{1}{\sqrt{2}}\widehat{j}$
(v) 0
(vi) $\frac{\pi}{2}$ rad

k)

(i) $|\underline{u}| = 10$
(ii) $|\underline{v}| = 10$
(iii) $\widehat{\underline{u}} = \frac{3}{5}\widehat{i} + \frac{4}{5}\widehat{j}$
(iv) $\widehat{\underline{v}} = -\frac{4}{5}\widehat{i} - \frac{3}{5}\widehat{j}$
(v) -96
(vi) 2.86 rad

l)

(i) $|\underline{u}| = \sqrt{41}$
(ii) $|\underline{v}| = \sqrt{34}$
(iii) $\widehat{\underline{u}} = \frac{5}{\sqrt{41}}\widehat{i} + \frac{4}{\sqrt{41}}\widehat{j}$
(iv) $\widehat{\underline{v}} = \frac{3}{\sqrt{34}}\widehat{i} + \frac{5}{\sqrt{34}}\widehat{j}$
(v) 35
(vi) 0.36 rad

m)

(i) $|\underline{u}| = \frac{1}{\sqrt{2}}$
(ii) $|\underline{v}| = \sqrt{29}$
(iii) $\widehat{\underline{u}} = \frac{1}{\sqrt{2}}\widehat{i} + \frac{1}{\sqrt{2}}\widehat{j}$
(iv) $\widehat{\underline{v}} = \frac{2}{\sqrt{29}}\widehat{i} + \frac{5}{\sqrt{29}}\widehat{j}$
(v) $\frac{7}{2}$
(vi) 0.40 rad

n)

(i) $|\underline{u}| = 4.70$
(ii) $|\underline{v}| = 4.00$
(iii) $\underline{\widehat{u}} = 0.87\widehat{i} + 0.49\widehat{j}$
(iv) $\underline{\widehat{v}} = -0.05\widehat{i} - 1.00\widehat{j}$
(v) -10.02
(vi) 2.13 rad

o)

(i) $|\underline{u}| = 4.42$
(ii) $|\underline{v}| = \sqrt{1+\pi^2}$
(iii) $\underline{\widehat{u}} = 0.79\widehat{i} + 0.61\widehat{j}$
(iv) $\underline{\widehat{v}} = \frac{1}{\sqrt{1+\pi^2}}\widehat{i} + \frac{\pi}{\sqrt{1+\pi^2}}\widehat{j}$
(v) $3.5 + 2.7\pi$
(vi) 0.61 rad

p)

(i) $|\underline{u}| = 1$
(ii) $|\underline{v}| = 1$
(iii) $\underline{\widehat{u}} = \frac{1}{\sqrt{2}}\widehat{i} + \frac{1}{\sqrt{2}}\widehat{j}$
(iv) $\underline{\widehat{v}} = \frac{1}{\sqrt{2}}\widehat{i} - \frac{1}{\sqrt{2}}\widehat{j}$
(v) 0
(vi) $\frac{\pi}{2}$ rad

6.

a)
(i) $\binom{4}{5}$
(ii) $\sqrt{41}$
(iii) $\frac{1}{41}\binom{4\sqrt{41}}{5\sqrt{41}}$

b)

(i) $\binom{-4}{-3}$
(ii) 5
(iii) $\binom{-\frac{4}{5}}{-\frac{3}{5}}$

c)

(i) $\binom{-1}{-1}$
(ii) $\sqrt{2}$
(iii) $\frac{1}{2}\binom{-\sqrt{2}}{\sqrt{2}}$

d)

(i) $\binom{-13}{-13}$
(ii) $13\sqrt{2}$
(iii) $\frac{1}{2}\binom{-\sqrt{2}}{\sqrt{2}}$

e)

(i) $-5\widehat{\mathbf{i}} + 3\widehat{\mathbf{j}}$
(ii) $\sqrt{34}$
(iii) $-\frac{5\sqrt{34}}{34}\widehat{\mathbf{i}} + \frac{3\sqrt{34}}{34}\widehat{\mathbf{j}}$

f)

(i) $-5\widehat{\mathbf{j}}$
(ii) 5
(iii) $-\widehat{\mathbf{j}}$

g)

(i) $2\widehat{\mathbf{i}} - 4\widehat{\mathbf{j}}$ (ii) $2\sqrt{5}$ (iii) $\frac{\sqrt{5}}{5}\widehat{\mathbf{i}} - \frac{2\sqrt{5}}{5}\widehat{\mathbf{j}}$

h)

(i) $\frac{1}{2}\widehat{\mathbf{i}} - \frac{3}{2}\widehat{\mathbf{j}}$ (ii) $\frac{1}{2}\sqrt{10}$ (iii) $\frac{\sqrt{10}}{10}\widehat{\mathbf{i}} - \frac{3\sqrt{10}}{10}\widehat{\mathbf{j}}$

7. a) $-\frac{8}{3}$ b) $\frac{2}{3}$ c) 25 d) $-\frac{12}{5}$ e) $\frac{3}{5}$ f) $\frac{4}{5}$ g) $\frac{5}{4}$ h) -12

8. a) 8 b) 1 c) -30 d) 75 e) 6 f) $\frac{5}{3}$ g) $\frac{5}{2}$ h) 10

9. (i) $\lambda = 0$ or 7 (ii) $5\sqrt{2}$ units

10.

a) (i) 63.4° (ii) 51.3° (iii) $\binom{3}{3}$ (iv) 45°

b) (i) 146.3° (ii) 56.3° (iii) $\binom{5}{1}$ (iv) 11.3°

c) (i) 36.9° (ii) 45° (iii) $\binom{-3}{-2}$ (iv) 213.7°

d) (i) 116.6° (ii) 51.3° (iii) $\binom{5}{3}$ (iv) 40.0°

e) (i) 306.9° (ii) 111.8° (iii) $-5\widehat{\mathbf{i}} + 9\widehat{\mathbf{j}}$ (iv) 119.1°

f) (i) 315° (ii) 63.4° (iii) $3\widehat{\mathbf{j}}$ (iv) 90°

g) (i) 303.7° (ii) 66.8° (iii) $\widehat{\mathbf{i}} + 10\widehat{\mathbf{j}}$ (iv) 84.3°

h) (i) 54.7° (ii) 35.3° (iii) $\frac{\sqrt{2}-1}{\sqrt{2}}\widehat{\mathbf{i}} + \frac{1-\sqrt{2}}{\sqrt{2}}\widehat{\mathbf{j}}$ (iv) 135°

11.

a) 420 km/h due North
b) 410 km/h due North
c) 475 km/h due North
d) 455 km/h due North
e) 450.25 km/h on bearing of 001.91°
f) 451.21 km/h on bearing of 004.19°
g) 450.31 km/h on bearing of 358.6°
h) 452.77 km/h on bearing of 353.66°

12.

a) 9 m/s due North East

b) 4 m/s due North East

c) 7 m/s due North East

d) 7.66 m/s on bearing of 049.76°

e) 7.14 m/s on bearing of 056.31°

f) 7.02 m/s on bearing of 040.91°

g) 8.33 m/s on bearing of 041.7°

h) 5.93 m/s on bearing of 054.36°