

## Revision Exercise (Differentiation I)

1.

a)  $\frac{dy}{dx} = 5x^4, \frac{d^2y}{dx^2} = 20x^3$

b)  $\frac{dy}{dx} = 18x^5, \frac{d^2y}{dx^2} = 90x^4$

c)  $\frac{dy}{dx} = 12x^2 + 2, \frac{d^2y}{dx^2} = 24x$

d)  $\frac{dy}{dx} = 1 - 5x^4, \frac{d^2y}{dx^2} = -20x^3$

e)  $\frac{dy}{dx} = -2x^3, \frac{d^2y}{dx^2} = -6x^2$

f)  $\frac{dy}{dx} = -\frac{3}{x^4}, \frac{d^2y}{dx^2} = \frac{12}{x^5}$

g)  $\frac{dy}{dx} = \frac{2}{5}, \frac{d^2y}{dx^2} = 0$

h)  $\frac{dy}{dx} = -\frac{7}{3x^2}, \frac{d^2y}{dx^2} = \frac{14}{3x^3}$

2.

a)  $f'(x) = \frac{3}{2}\sqrt{x}, f''(x) = \frac{3}{4\sqrt{x}}$

b)  $f'(x) = \frac{5}{2}\sqrt{x^3}, f''(x) = \frac{15}{4}\sqrt{x}$

c)  $f'(x) = 7\sqrt{x^5}, f''(x) = \frac{35}{2}\sqrt{x^3}$

d)  $f'(x) = \frac{1}{\sqrt[3]{x^2}}, f''(x) = -\frac{2}{3\sqrt[3]{x^5}}$

e)  $f'(x) = -\frac{2}{\sqrt{x}}, f''(x) = \frac{1}{\sqrt{x^3}}$

f)  $f'(x) = -\frac{3}{2\sqrt{x^3}}, f''(x) = \frac{9}{4\sqrt{x^5}}$

g)  $f'(x) = \frac{2}{5\sqrt[5]{x^6}}, f''(x) = -\frac{12}{25\sqrt[5]{x^{11}}}$

h)  $f'(x) = \frac{5}{2\sqrt{x^7}}, f''(x) = -\frac{35}{4\sqrt{x^9}}$

3.

a)  $2x + 3x^2$

c)  $\frac{7}{2} + 10x^4$

e)  $\frac{1}{2\sqrt{x}} - \frac{20}{x^6}$

g)  $\frac{10}{3\sqrt[3]{x}} + \frac{1}{\sqrt{x^3}}$

b)  $4x + \frac{1}{x^2}$

d)  $-\frac{1}{x^2} + \frac{6}{x^4}$

f)  $-\frac{1}{2\sqrt{x^3}} - \frac{3}{\sqrt{x^5}}$

4.

a)  $40(4x + 1)^9$

d)  $4 \cos(4x - \frac{\pi}{2})$

g)  $6 \sin^2 2x \cos 2x$

j)  $-\frac{6x^2}{\sqrt{5-4x^3}}$

m)  $\frac{10}{3\sqrt[3]{5x-2}}$

b)  $-6(3 - 2x)^2$

e)  $-\frac{1}{2} \sin \frac{x}{2}$

h)  $6\sqrt{4x+1}$

k)  $\frac{1}{3\sqrt[3]{(x+2)^2}}$

n)  $\frac{3}{4\sqrt{4x+1}}$

c)  $\frac{3}{2\sqrt{3x+5}}$

f)  $20 \cos 2x$

i)  $\frac{2x}{\sqrt{2x^2-5}}$

l)  $\frac{2x}{3\sqrt[3]{(3x^2+1)^2}}$

o)  $\frac{3}{4}\sqrt{1+\frac{x}{2}}$

5.

a)  $-18(1 - 3x)^5$

h)  $\sqrt{x-2} + \frac{1}{2}\frac{x}{\sqrt{x-2}}$

b)  $15(3x + 4)^4$

i)  $3\sqrt{2-3x} - \frac{9}{2}\frac{x}{\sqrt{2-3x}}$

c)  $2(x+2)(4x-2)^3 + 12(x+2)^2(4x-2)^2$

j)  $12x^2 \sin 2x + (8x^3 + 4) \cos 2x$

d)  $2 \sin 2x + 4x \cos 2x$

k)  $-6x \cos 4x - 4(1 - 3x^2) \sin 4x$

e)  $9x^2 \cos^2 2x - 12x^3 \cos 2x \sin 2x$

l)  $12(4x+3)^2 \sin x + (4x+3)^3 \cos x$

f)  $6x \sin(3x-2) + 3(3x^2+2) \cos(3x-2)$

m)  $-20(4-5x)^3 \cos x - (4-5x)^4 \sin x$

g)  $\frac{2}{x^2} + 6x - 18(2x+5)^2$

6.

a)  $-\frac{18}{(2x-3)^2}$

b)  $-\frac{1}{(x+1)^2}$

c)  $-\frac{6}{(x+2)^2}$

d)  $\frac{25}{(x+3)^2}$

e)  $\frac{x^2+6x-2}{(x+3)^2}$

f)  $-\frac{x+4}{\sqrt{(x+1)^3}}$

g)  $\sec^2 x$

h)  $-\operatorname{cosec}^2 x$

i)  $\frac{3x \cos 3x + 3 \cos 3x - \sin 3x}{(x+1)^2}$

j)  $\frac{\cos x - 2x \cos x + 2 \sin x}{(1-2x)^2}$

7.

a) Minimum at  $(\frac{3}{2}, -\frac{9}{4})$

b) Maximum at  $(2, 7)$

c) Minimum at  $(\frac{3}{4}, \frac{23}{8})$

d) Maximum at  $(-2, 7)$ ; Minimum at  $(0, 3)$

e) Maximum at  $(1, 11)$ ; Minimum at  $(2, 10)$

f) Maximum at  $(-3, 18)$ ; Minimum at  $(3, -18)$

g) Minimum at  $(-2, -64)$ ; Maximum at  $(2, 64)$

h) Minima at  $(0, 1)$  and  $(2, 1)$ ; Maximum at  $(1, 2)$

8.

a) Tangent:  $y = 4x - 1$ ; Normal:  $y = -\frac{1}{4}x + \frac{13}{4}$

b) Tangent:  $y = 4x - 15$ ; Normal:  $y = -\frac{1}{4}x + 2$

c) Tangent:  $y = 2x + 4$ ; Normal:  $y = -\frac{1}{2}x + \frac{3}{2}$

d) Tangent:  $y = 5x - 3$ ; Normal:  $y = -\frac{1}{5}x + \frac{11}{5}$

e) Tangent:  $y = \frac{1}{4}x + 1$ ; Normal:  $y = -4x + 18$

f) Tangent:  $y = -\frac{1}{4}x + 1$ ; Normal:  $y = 4x - \frac{15}{2}$

g) Tangent:  $y = x + 1$ ; Normal:  $y = -x - 1$

h) Tangent:  $y = 10x + 9$ ; Normal:  $y = -\frac{1}{10}x - \frac{11}{10}$

i) Tangent:  $y = x$ ; Normal:  $y = -x$

j) Tangent:  $y = -\pi x + \pi^2$ ; Normal:  $y = \frac{1}{\pi}x - 1$

k) Tangent:  $y = 1$ ; Normal:  $x = -2$

9.

a) Gradient of first tangent at  $(0, 2)$  is 3. Gradient of second tangent at  $(-3, 2)$  is  $-3$ .

b) Equation of first tangent is  $y = 3x + 2$ . Equation of second tangent is  $y = -3x - 7$ .

c) Equation of first normal is  $y = -\frac{1}{3}x + 2$ . Equation of second normal is  $y = \frac{1}{3}x + 3$ .

10.

a)  $b = 2$  and  $c = 3$

b)  $b = 12$  and  $c = 22$

11.

a)  $x = 0$

b)  $x = -\frac{2}{5}$

c)  $x = 0$  or  $-3$

d)  $x = -2$  or  $-3$